ASSIGNMENT-2

① Let $X_1, X_2, \cdots, X_n$ be a random sample from a distribution having p.d.f.

$$f(x;\theta) = \frac{\{x(1-x)\}^{\theta-1}}{Beta(\theta,\theta)} \; ; \; 0<x<1, \theta>0$$

Show that the best critical region for testing $H_0: \theta=1$ against $H_1: \theta=2$ is

$$W = \left\{ (x_1, x_2, \cdots, x_n) : c \le \sum_{i=1}^{n} x_i(1-x_i) \right\}.$$

② It is required to test the null hypothesis

$H_0: f(x) \sim \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \; -\infty < x < \infty$

against $H_1: f(x) \sim \frac{1}{2} e^{-|x|}; \; -\infty < x < \infty$.

Obtain the most powerful test.

③. The p.d.f. of X is given by $f(x) = \frac{1}{\theta}, \; 0<x<\theta$.

Let the null hypothesis be $H_0: \theta = \frac{4}{3}$ against the alternative $H_1: \theta = 5/3$. Based on one observation construct a MP critical region.

④ Examine whether a best critical region exists for testing the null hypothesis $H_0: \theta=\theta_0$ against $H_1: \theta=\theta_1$ for the parameter θ of the distribution

$$f(x;\theta) = \frac{1+\theta}{(x+\theta)^2} \; ; \; 1 \le x < \infty.$$

(Based on one observ)

⑤ p is the probability that a given die shows even number. To test $H_0: p=\frac{1}{2}$ against $H_1: p=\frac{1}{3}$, following procedure is adopted. Toss the die twice and accept H_0 if both times it shows even number. Find the probability of type I error and prob. of type II error.

⑥ Let X have p.d.f. $f(x) = \frac{1}{\theta} e^{-x/\theta}, \; 0<x<\infty, \theta>0$. $H_0: \theta=2$ against $H_1: \theta=1$ use a random sample X_1 and X_2 and define $W = \{(x_1, x_2) : 9.5 \le x_1+x_2\}$

Find power and level of significance.